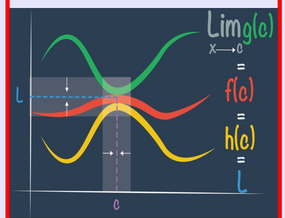
Math 261

Fall 2022

Lecture 30



Given $f(x) = 2 + 3x^2 - x^3$

$f(0) = 2$ ✓

$f(x)$ is polynomial

Domain $(-\infty, \infty)$

Cont. everywhere

$f'(x) = 6x - 3x^2$

$f'(x) = 3x(2 - x)$

$f'(x) = 0 \rightarrow x = 0 \quad x = 2$

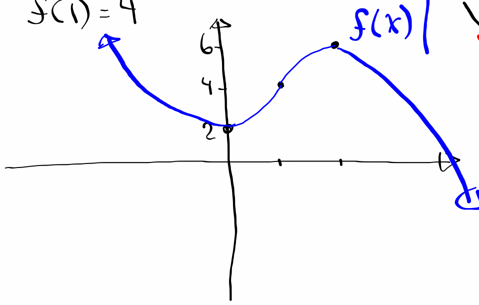
$f(0) = 2$ ✓, $f(2) = 6$

$f''(x) = 6 - 6x$

$f''(x) = 0 \rightarrow x = 1$

$f(1) = 4$

x	$-\infty$		0		1		2		∞
$f'(x)$		$-$	•	$+$		$+$	•	$-$	
$f''(x)$		$+$		$+$	•	$-$		$-$	
$f(x)$			Min				Max		



only one
x-Int

$f(x) = \frac{1}{x^2-9}$ → $x^2-9 \neq 0$ $\quad x \neq \pm 3$

Domain: $(-\infty,-3)\cup(-3,3)\cup(3,\infty)$

V.A. $x=3$, $x=-3$

No x-Int since $f(x) \neq 0$

Y-Int $(0, \frac{-1}{9})$

$f(-x) = \frac{1}{(-x)^2-9} = \frac{1}{x^2-9} = f(x)$

$f(-x) = f(x)$ → even Function

Symmetric to Y-axis.

$f(x) = (x^2-9)^{-1}$

$f'(x) = -1(x^2-9)^{-2} \cdot 2x = \frac{-2x}{(x^2-9)^2}$

$f'(x) = 0$ → $x = 0$

$f'(x)$ undefined at $x = \pm 3$

$f'(x) = -2x(x^2-9)^{-2}$ $\quad f''(x) = -2\left[1(x^2-9)^{-2} + x \cdot -2(x^2-9)^{-3} \cdot 2x\right]$

$f''(x) = -2(x^2-9)^{-3}\left[(x^2-9)^1 - 4x^2\right]$

$f''(x) = \frac{-2(-3x^2-9)}{(x^2-9)^3} = \frac{6(x^2+3)}{(x^2-9)^3}$

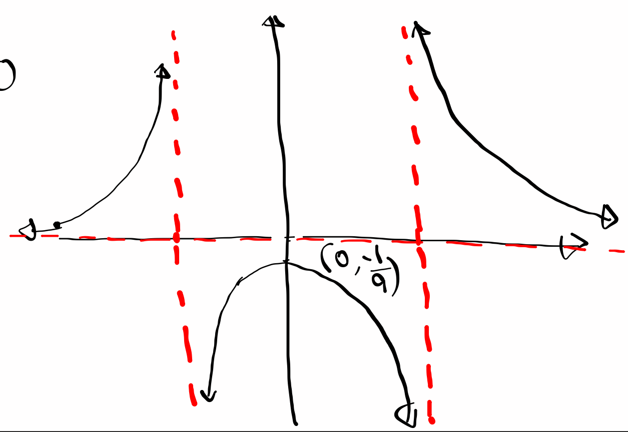
$f''(x) \neq 0$, $f''(x)$ is undefined @ $x = \pm 3$.

x	$-\infty$		-3		0		3		∞
$f'(x)$		+	o	+	•	−	o	−	
$f''(x)$		+	o	−		−	o	+	
$f(x)$									

$\lim_{x\to\pm\infty} f(x) = \lim_{x\to\pm\infty} \frac{1}{x^2-9} = 0$

Range

$(-\infty, \frac{-1}{9}] \cup (0,\infty)$



$f(x)=\frac{x^2}{x^2+9}$ $\qquad f(0)=\frac{0}{9}=0$ $\qquad f(-x)=f(x)$

Domain: $(-\infty,\infty)$ $\qquad$ x-Int $(0,0)$ $\qquad$ even function

Y-Int $(0,0)$ $\qquad$ Symmetry → Y-axis

$\lim_{x\to\pm\infty} f(x)=1$ $\qquad$ Find $f'(x)$

H.A. $y=1$

$f(x)=\frac{x^2+9-9}{x^2+9}=\frac{x^2+9}{x^2+9}-\frac{9}{x^2+9}$

$f(x)=1-\frac{9}{x^2+9}$ $\qquad f(x)=1-9(x^2+9)^{-1}$

$f'(x)=-9\cdot(-1)(x^2+9)^{-2}\cdot 2x \to f'(x)=18x(x^2+9)^{-2}$

$f'(x)=0 \to x=0$

$f''(x)=18\left[1(x^2+9)^{-2}+x\cdot -2(x^2+9)^{-3}\cdot 2x\right]$

$=18(x^2+9)^{-3}\left[(x^2+9)^1-4x^2\right]=18(x^2+9)^{-3}(9-3x^2)$

$f'(x)=\frac{18x}{(x^2+9)^2}$ $\qquad f''(x)=\frac{-54(x^2-3)}{(x^2+9)^3}$

$x=0$ $\qquad x=\pm\sqrt{3}$

x	$-\infty$ to $-\sqrt{3}$	$-\sqrt{3}$ to 0	0 to $\sqrt{3}$	$\sqrt{3}$ to ∞
$f'(x)$	−	−	+	+
$f''(x)$	−	+	+	−
$f(x)$	↘	↘	↗	↗

I.P. $(\pm\sqrt{3},\frac{1}{4})$ $\qquad$ C.P. $(0,0)$

No V.A. , H.A. $y=1$ $\qquad f(x)\geq 0$